STIFFENING OF THIN CYLINDRICAL SILO SHELL AGAINST BUCKLING LOADS

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ABSTRACT

Buckling is one of the important criteria to be checked in silo design. Buckling behaviour of silos can be analyzed with finite element methods or analytical methods which are developed for cylindrical shells.

In this study, buckling analysis of a silo which is also supporting another structure is analyzed with eigenvalue solution by using finite element method and one of these analytical methods. The results obtained by using these two methods are compared.

Keywords: Buckling, cylindrical shells, eigenvalue, finite element method.

1. INTRODUCTION

Containers for the storage of bulk solids are usually called bins, bunkers and silos. Steel silos in common use are usually in circular in cross section and may be ground-supported or elevated. In practice, typical elevated silos are designed as three main sections: plain roof, cylindrical shell and conical hopper.

Basically, cylindrical silo walls are subjected to both normal pressures and vertical frictional shears or tractions due to stored material inside the silo which vary along the wall. The normal pressures on cylindrical walls will cause circumferential hoop stress; and the vertical frictional shear will cause cumulative axial compressive stress.

Although many empirical approximations and different theories have been developed to predict the pressures on the vertical walls of a silo, Janssen theory is the one which is widely accepted [Janssen, 1895]. Janssen's equation was used by Pieper and Wenzel for calculation of maximum flow pressures on the vertical walls of the silo by using lateral pressure ratio "k" and wall friction coefficient "μ" (Hongyu, [1994]). This approach was also used in DIN 1055 – Part 6 [1964, 1987] standard.

Axial compressive load due to the stored material which was mentioned above, together with wind, seismic loads and other torsion and bending moments, axial forces which are coming from dead weight of silo and the weights supported by silo are the main causes for shell buckling of silos.

The first theoretical solutions for the buckling strength of cylinders under the axial loading were presented by Lorenz (1908), Timoshenko (1910) and Southwell (1914). These solutions were restricted to typical perfect cylinders with simple boundary conditions and assumed a uniform membrane stress state on elastic materials prior to buckling. A uniform membrane prebuckling stress distribution which is commonly named as the classical elastic axial buckling stress, was expressed by Timoshenko (Timoshenko & Gere [1961]) as:

$$\sigma_{cl} = (E.t) / (r.[3\ (1 - v^2)]^{1/2}) \qquad (1)$$

Where, E is the Young's modulus, t is the membrane thickness, r is the shell radius and v is the Poisson's ratio.

Although there are many limitations, the classical elastic buckling stress is commonly used as reference value for buckling calculations. Studies for the effects of various boundary conditions in buckling analysis are still on going.

Due to the complexity of the problem, the finite element and numerical integration techniques are very widely used for buckling and collapse analyses. However, there are very few computer programs specifically designed for silos. Therefore, general shell finite element analysis (FEA) programs are commonly used.

To assess the buckling behaviour of a thin cylindrical silo shell, this paper employs two techniques. One of them is eigenvalue solution by FEA and the other one is analytical method according to Det Norske Veritas DNV-RP-C202 [2002] which allows checking buckling stability of unstiffened and stiffened cylindrical shells.

2. SILO DESIGN

In this study, buckling check was performed for a clinker silo which was installed for Line 6 of YAMAVER 5 Cement Factory in Saudi Arabia.

2.1. Characteristics of Silo: Figure 1 and Figure 2 show sectional and elevation views of silo. Due to customer requirements, diameter, shell height and hopper height of the silo are 7000 mm, 16410 mm and 3200 mm respectively. Clinker with density of 14 kN/m^3 is stored in this silo and it is supported by skirt on concrete supports. The final shell thickness for silo, which was reached after analysis, is between 8 mm and 16 mm as shown on Figure 1 (Corrosion allowance of 1.5 mm is specified in the specifications. Therefore, shell thickness less this corrosion allowance figure was used in analyses). This silo is also supporting another steel structure on top of its roof to which belt conveyor lines are connected.

2.2. Loads Acting to Silo: Table 1 and Figure 3 show silo loads coming from the stored material which are calculated as per DIN 1055 - Part 6 [1987]. For filling case, loads are indicated with “fill” subscript and for discharging case, loads are indicated with “disc” subscript. Since the loads are higher for discharging case, the figures for this case were used for the analyses.

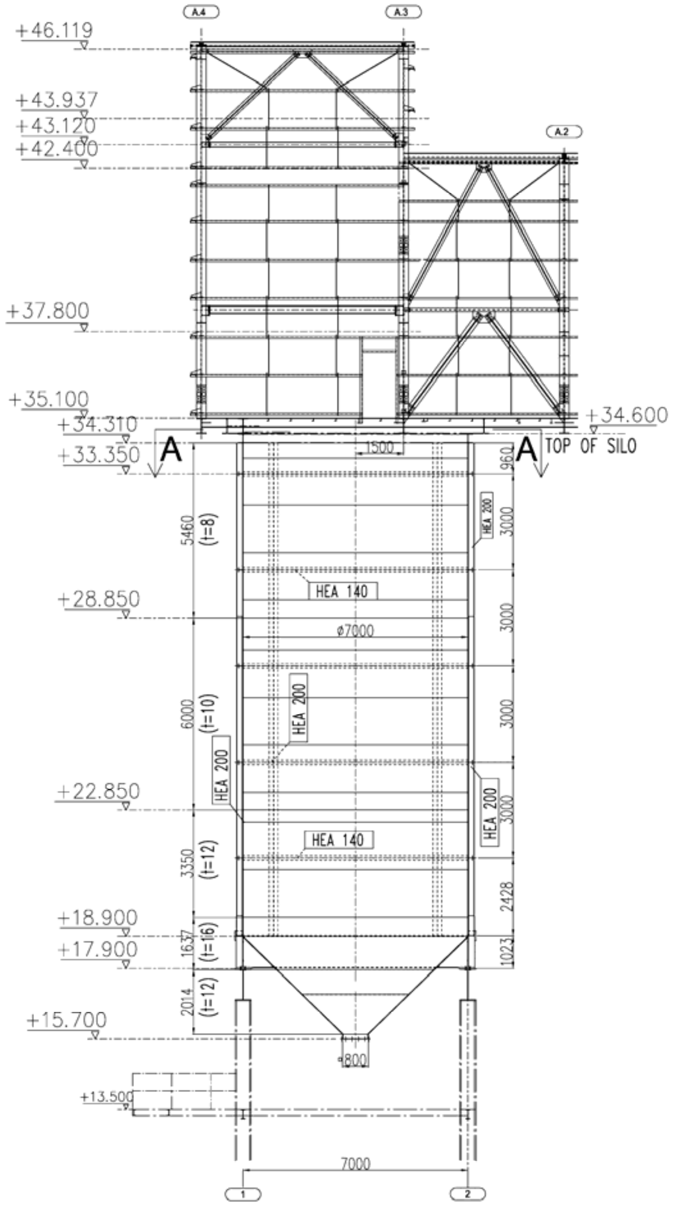


Figure 1. Silo Elevation View

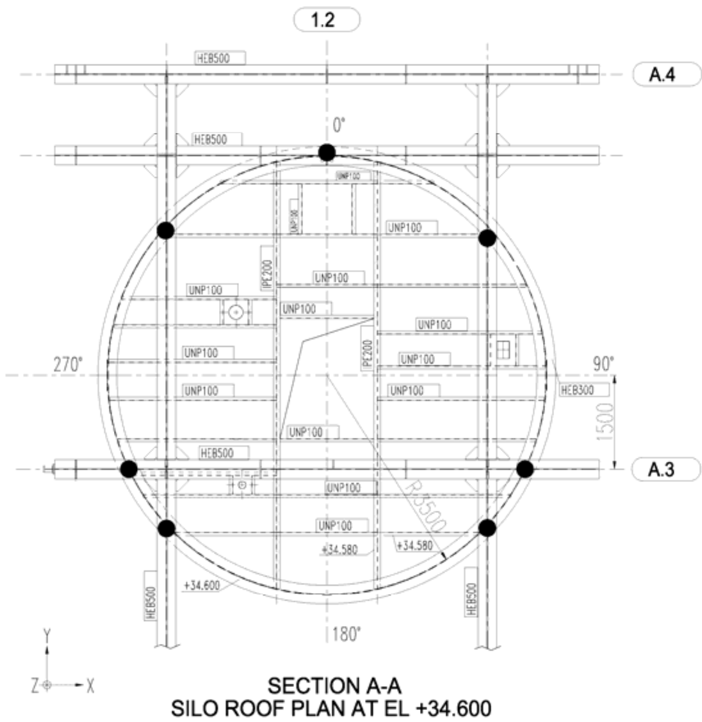


Figure 2. Sectional View at Silo Roof

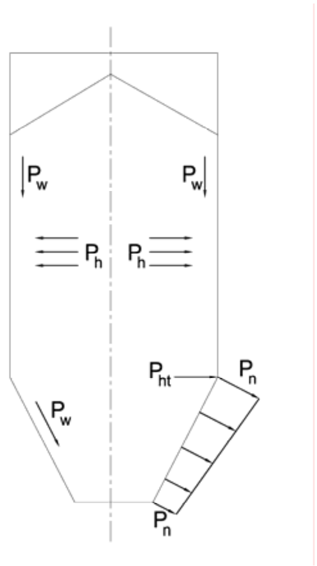


Figure 3. DIN 1055 – Part 6 Loads Legend

Since the silo is supporting another steel structure on top of it, there are base reactions of this structure on silo roof which are acting at points indicated on Figure 2, Silo Roof Plan. Summation of the base reaction values which are acting as building load to the silo roof are listed in Table 2.

This silo is exposed to wind loads as well. The specified wind load for the region where the silo was constructed is 1.54 kPa. Wind load is acting partially as external pressure and partially as suction load. In analytical method, suctional wind load is included as external pressure with the factor indicated in DIN 1055 – Part 4 [1987]. In the FEA model, the wind load is simulated as per actual wind load pattern. Due to the regional conditions, where the silo was constructed, no seismic load was taken into consideration for analyses.

Table 1. Silo Loads as per DIN 1055 – Part 6 [1987]

Silo Part	Thk. mm	Ph_{fill} kPa	Ph_{disc} kPa	Pn_{fill} kPa	Pn_{disc} kPa	Pw_{hfill} kPa	Pw_{hdisc} kPa	Pw_{fill} kN/m	Pw_{disc} kN/m
Cylindrical	8	27.5	33	_	_	_	_	37.7	41.4
Cylindrical	10	42	50.4	_	_	_	_	133.9	147.3
Cylindrical	12	46.3	125.6	_	_	_	_	200.7	220.7
Cylindrical	16	46.9	154.3	_	_	_	_	213.3	234.6
Hopper	16	_	_	167.9	283.4	70	76	_	_
Hopper	12	_	_	84.5	84.5	54	56	_	_

Table 2. Base Reactions of Building on Silo Roof

Fx kN	Fy kN	Fz kN	Mx kNm	My kNm	Mz kNm
487	0	936	520	482	5

3. LINEAR BUCKLING ANALYSIS WITH FEA MODEL

3.1. Linear Buckling Analysis: Linear buckling analysis seeks the instability modes of a structure due to P-delta effect under axial loads, silo wall frictions and external (wind) pressures. Buckling analysis involves the solution of the generalized eigenvalue problem which is expressed as:

$$([K] - \lambda\,[G])\,\{\psi\} = 0 \tag{2}$$

Where, K is the elastic stiffness matrix of the structure, G is the geometric stiffness matrix of the structure resisting unit load, λ is critical load factor and ψ is the matrix of eigenvectors (ultimate deformation vector) of the structure.

In Equation (2), the matrix of eigenvector is not equal to zero ($\{\psi\} \neq 0$). Therefore,

$$|\,[K] - \lambda\,[G]\,| = 0 \tag{3}$$

In this case, the buckling analysis becomes eigenvalue problem. The diagonal matrix of eigenvalues (λ) is called as buckling factor. That is, it is a safety factor against buckling under the applied loads

The buckling modes depend on the loads, and it is recommended that at least six buckling modes were checked (CSI, [2002]). In order to take large displacement and tensile or compressive direct stress into consideration, initial nonlinear P-Delta analysis is carried out and the stiffness matrix obtained from this P-Delta analysis is used for linear buckling analysis for each set of loads at concern.

Resulting eigenvalues, which are calculated by linear buckling analysis for different buckling modes, give different safety factors against buckling. These factors should be greater than one for stability against buckling. Negative eigenvalues indicate that buckling will occur if the loads are reversed.

3.2. Finite Element Program: In this study, the silo buckling behaviour was investigated with linear buckling analysis of SAP 2000 (Nonlinear v.8.3.4)**.** SAP 2000 is a general purpose structural finite element analysis program. It is not possible to simulate flow and other time dependent dynamic effects by SAP 2000 (Nonlinear v.8.3.4). Therefore, static analysis for the most critical loading cases was carried out.

3.3. Modeling of the Silo Structure: In this study, two silo FEA models were developed. One of them, which is named as SILO1, had no stiffening frame elements on silo wall. The other silo model, which is named as SILO2, was orthogonally (both circumferentially and longitudinally) stiffened on silo wall. Both of them were checked for buckling stability.

These silo shells were modeled by using thin shell elements with six degrees of freedom; and shell material was assumed to be linear, isotropic and in S235JRG2 (DIN EN 10025 [1994]) steel grade with Young's modulus of E = 210 GPa and Poisson's ratio of v = 0.3. Generation of shell meshes was made by dividing circumference of the shell into 36 pieces and dividing the shell lengths to give approximately square meshes with 0.38% roundness error. All circumferential and longitudinal stiffeners were modeled as frame elements with steel grade of S235JR (DIN EN 10025, [1994]). SILO1 model consists of 2132 shell and 182 frame elements and SILO2 model consists of 2132 shell and 614 frame elements.

3.4. Performed Linear Buckling Analyses and Results : The linear buckling analyses by using initial P-Delta analysis were performed for loading cases 1 through 4 which are given in Table 3.

Nonlinear P-Delta analyses were carried out with P-Delta plus large displacement geometric nonlinear parameters for maximum 200 total steps and maximum 50 null steps per stage with 1.10^{-7} iteration convergence tolerance. Linear buckling analyses were carried out for 6 buckling modes and 1.10^{-9} eigenvalue convergence tolerance.

Since eigenvalues for all loading cases (CASE 1 through CASE 4) are less than one at least for some modes as shown in Table 4, the silo is not stable against buckling in its unstiffened form (SILO1).

In Figure 4 different buckling mode shapes are indicated for loading CASE 1, which gives the lowest buckling stability. It is apparent from this figure that buckling is affecting the silo shell starting from top to almost bottom.

There are two practical methods to eliminate this buckling behaviour. One of them is to increase the silo shell thickness and the other one is stiffening the silo shell with frame elements.

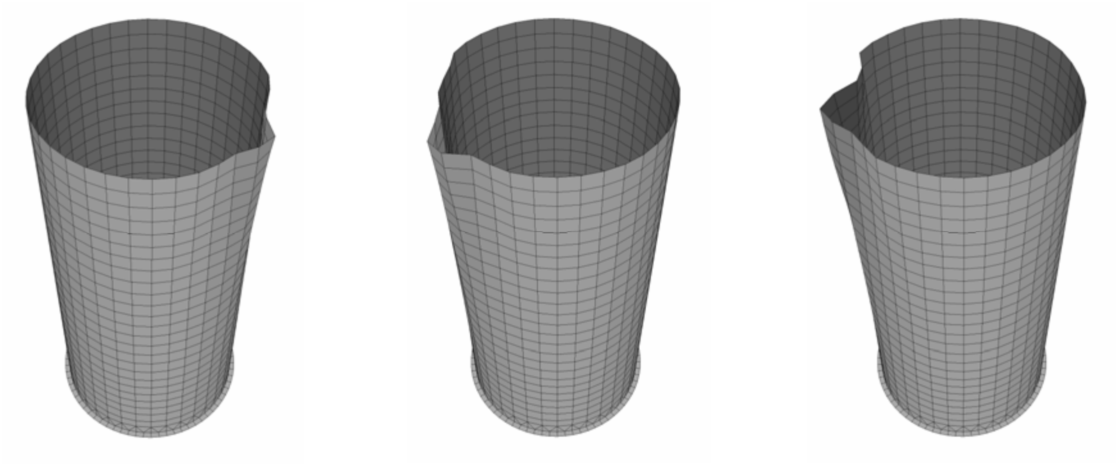
Table 3. Finite Element Analysis Cases

Case Name	Loads	
	Initial P-Delta Analysis	Linear Static Buckling Analysis
CASE 1	PW_{disc}, WIND, DEAD	BUILDING
CASE 2	PW_{disc}, BUILDING, DEAD	WIND
CASE 3	DEAD	WIND, BUILDING
CASE 4	PW_{disc}, DEAD	WIND, BUILDING

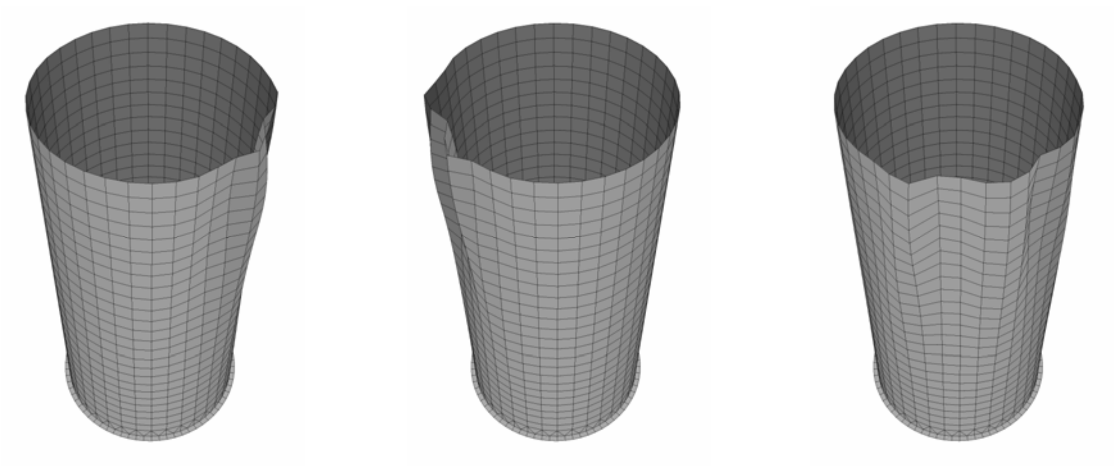
Table 4. FEA Linear Buckling Analysis Results of SILO1 for Different Buckling Modes

Case	Results of Eigenvalue					
Name	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
CASE1	0.2	-0.3	-0.4	0.5	-0.6	0.7
CASE2	0.5	-0.5	-0.7	0.7	-0.7	-0.8
CASE3	0.7	0.8	1.1	1.2	1.3	-1.3
CASE4	3.5	3.8	-3.8	3.9	3.9	-4.5

The latter option was preferred because of its weight (consequently cost) advantage. Therefore, the silo was reinforced with eight HEA200 members longitudinally and with 3000 mm spaced HEA140 members circumferentially. Also, shell–cone joint was circumferentially reinforced with T member (T300x40-190x25) and shell–roof joint was circumferentially reinforced with HEB300 members (Figure 1, Figure 2).



a. Mode 1 b. Mode 2 c. Mode 3



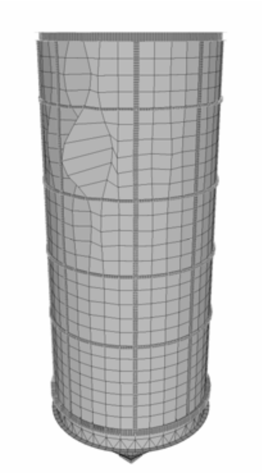
d. Mode 4 e. Mode 5 f. Mode 6

Figure 4. Buckling Modes of SILO1 for CASE 1 (Sections at EL.+33.14 m-Scale 30000x)

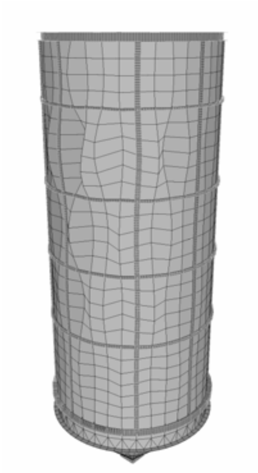
Table 5. FEA Linear Buckling Analysis Results of SILO2 for Different Buckling Modes

Case Name	Results of Eigenvalue					
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
CASE1	34.8	35.9	38.3	38.5	38.9	-39.1
CASE2	22.9	25.5	34.8	43.1	43.9	47.6
CASE3	16.5	18.6	20.9	25.6	26.5	28.3
CASE4	13.7	14.7	16.2	16.9	17.3	17.8

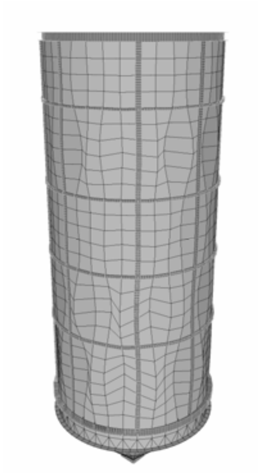
Eigenvalues for this orthogonally stiffened model (SILO2) are given in Table 5. Since all eigenvalues for this model are greater than one, this implies that the orthogonally stiffened silo is stable against buckling. The resulting buckling mode shapes for this stiffened silo model under load CASE 4 is given in Figure 5.



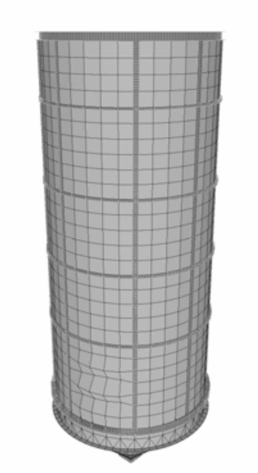
a. Mode 1



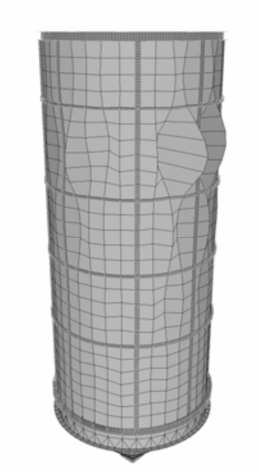
b. Mode 2



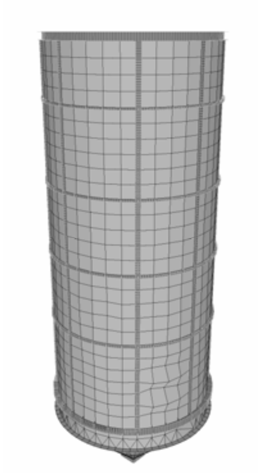
c. Mode 3



d. Mode 4



e. Mode 5



f. Mode 6

Figure 5. Buckling Modes of SILO2 for CASE 4 (Scale 30000x)

Linear buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure. However, imperfections and nonlinearities such as residual stresses in fabricated structures prevent actual structures from achieving their theoretical classic buckling strength. Therefore, linear buckling analysis often gives unconservative results (ANSYS, [2004]). Thus, nonlinear finite element buckling analysis is recommended for achieving conservative stability solutions.

Since such analysis can only be performed with advanced finite element programs, and since realistic definitions for the above mentioned imperfections and nonlinearity to this programs are time consuming and difficult, conservative analytical methods are preferred for stability analysis of silo type structures.

Therefore, the results obtained by linear buckling analysis for stiffened silo are checked and verified with one of these analytical methods for final design.

4. ANALYTICAL METHOD FOR BUCKLING ANALYSIS

Among various analytical buckling analysis methods, the method given in "Recommended Practice DNV-RP-C202" of DNV [2002] was selected because of its applicability for both unstiffened and stiffened cylindrical shells.

This recommended practice treats the buckling stability of shell structures based on the load and resistance factor design format (LRFD). But it may also be used with working stress design format (WSD) by some substitutions mentioned in this recommended practice.

The buckling analysis methods in this recommended practice are to be considered as semi-empirical; because theoretical and experimental buckling loads for some cases have been found out that inconsistent. This discrepancy is due to the effect of geometric imperfections and residual stresses in fabricated structures. Semi-empirical methods of this recommended practice take into account these imperfections up to an assumed level.

In this practice, buckling mode checks which are performed for different stiffening conditions are indicated in Table 6.

Table 6. Buckling Mode Checks for Different Stiffening Conditions per DNV-RP-C202

Buckling Modes	Stiffening Conditions			
	Unstiffened	Ring Stiff.	Longitudinal Stiff.	Orthogonally Stiff.
Cylindrical Shell	YES	YES	NO	NO
Curved Panel	NO	NO	YES	YES
Panel Ring	NO	YES	NO	YES
Panel Stiffener	NO	NO	YES	YES
General	NO	NO	NO	YES
Column	YES	YES	YES	YES

Shell courses of silo with different thickness values are checked separately for buckling modes with unstiffened, ring stiffened and orthogonally stiffened conditions under given loads by using the method of DNV-RP-C202 [2002]. Flow chart which summarizes the buckling checking procedure of DNV-RP-C202 is given in Figure 6.

The results obtained with this analysis are listed in Table 7. The following notation is used for Table 7:

$$S\text{: Buckling Safety Factor} = \frac{\text{Design Buckling Strength } (f_{ksd})}{\text{Design Equivalent von Mises' Stress } (\sigma_{j,sd})}$$

or

$$= \frac{\text{Area } (A_R) \text{ or Eff. Inertia } (I_{eff}) \text{ of ring stiffener}}{\text{Required Area } (A_{Req}) \text{ or Inertia } (I_R) \text{ of ring stiffener}}$$

or

$$= \frac{\text{Column Buckling Strength}}{\text{Design Compression Stress}} \qquad (\text{if } [kL_c / i_c]^2 \geq 2.5\ E/f_y)$$

Subscripts “sh” stands for cylindrical shell or curved panel, “ring” stands for panel ring stiffener, “stiff” stands for panel stiffener and “col” stands for column.

Since buckling stability is achieved with orthogonally stiffened silo shell (where safety factors are greater than one), the final design was completed for this stiffening configuration; and fabricated silo according to this design have been installed for Line 6 of YAMAVER 5 Cement Factory in Saudi Arabia as shown at right hand side of Figure 7.

Table 7. Buckling Stability Results of Analytical Method for Cylindrical Part of the Silo.

Shell Thk. (mm)	Stiffening Condition									
	Unstiffened		Ring Stiffened[1]			Orthogonally Stiffened[2]				
	S_{sh}	S_{col}	S_{sh}	S_{ring}	S_{col}	S_{sh}	S_{stiff}	S_{ring}	Gen.[3]	S_{col}
8	0.61	N/A	0.98	2.58	N/A	1.15	2.31	2.58	YES	N/A
10	0.82	N/A	1.14	1.97	N/A	1.31	3.12	1.97	YES	N/A
12	1.17	N/A	1.32	1.60	N/A	1.50	3.55	1.60	YES	N/A
16	2.49	N/A	2.49	1.40	N/A	2.69	5.87	1.40	YES	N/A

1. Stiffening rings are HEA140 with 3000 mm spacing.
2. Stiffening rings are HEA140 with 3000 mm spacing and longitudinal stiffeners are HEA200 at each 45°.
3. Gen.: Satisfaction of stiffener geometric proportions.

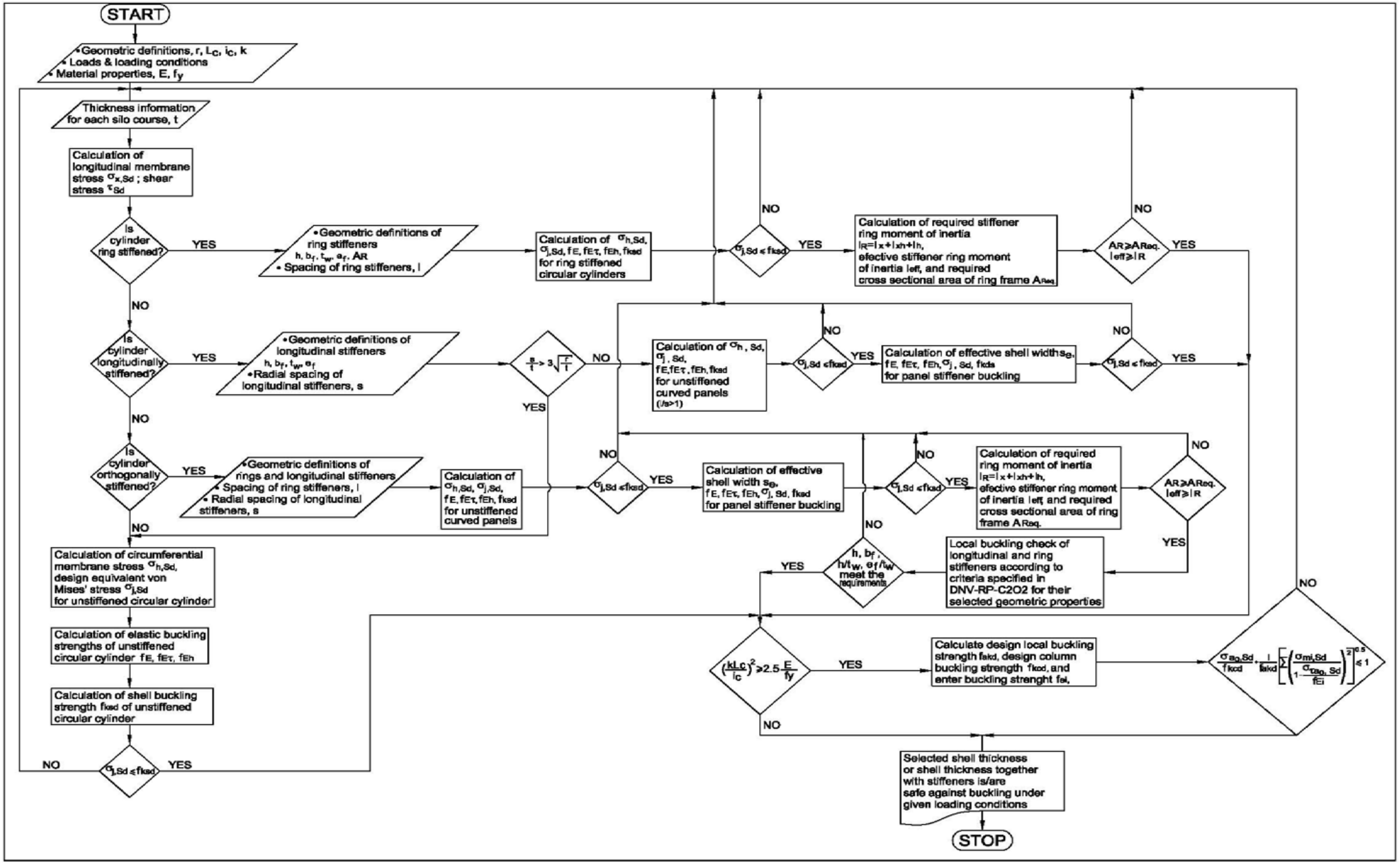


Figure 6. Flowchart for Analytical Solution per DNV-RP-C202

a. During Erection Stage　　　　b. At the Final Stage

Figure 7. Site Photos for Silos

5. CONCLUSION

In this study, buckling check was performed with linear buckling analysis method and analytical method for a silo. Since hopper part of the silo is not critical for buckling due to the existing loads, analytical buckling analysis for this part was omitted.

During design phase, the most cost effective solution was tried to be reached and it was concluded that stiffening of shells gives lighter solutions instead of increasing shell thickness.

It is found out that the linear buckling analyses with finite element method gives less conservative results (Table 5) compared to analytical methods (Table 7) which takes into consideration for geometric imperfections and residual stresses in fabricated structures as specified in various literature.

Therefore, it is recommended to use analytical methods which consider the above mentioned secondary effects in stead of linear buckling analysis, when nonlinear buckling analysis with realistic initial conditions can not be performed.

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